Math 1B
Quiz 8
Fri Nov 30, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: _____ / 30 POINTS

1. Show organized & proper work (including proper notation)
2. If an improper integral diverges, write "DIVERGES"

Evaluate $\int_e^\infty \frac{dx}{x(\ln x)^3}$.

SCORE: ___/ 7 PTS

$= \lim_{N\to\infty} \int_e^N \frac{dx}{x(\ln x)^3}$ (1) (1) (1/2)

$u = \ln x$, $du = \frac{dx}{x}$ (1)

$\int \frac{du}{u^3} = -\frac{1}{2u^2} = -\frac{1}{2(\ln x)^2}$

$= \lim_{N\to\infty} -\frac{1}{2(\ln x)^2}\Big|_e^N$ (1 1/2)

$= \lim_{N\to\infty} \left(\frac{1}{2(\ln N)^2} + \frac{1}{2}\right)$ (1)

$= 0 + \frac{1}{2}$

$= \frac{1}{2}$ (1)

Evaluate $\int_0^9 \frac{dx}{\sqrt[3]{x-1}}$.

SCORE: ___/ 8 PTS

$= \int_0^1 \frac{dx}{\sqrt[3]{x-1}} + \int_1^9 \frac{dx}{\sqrt[3]{x-1}}$

$= \lim_{N\to 1^-} \int_0^N (x-1)^{-\frac{1}{3}} dx + \lim_{M\to 1^+} \int_M^9 (x-1)^{-\frac{1}{3}} dx$

$= \lim_{N\to 1^-} \frac{3}{2}(x-1)^{\frac{2}{3}}\Big|_0^N$ (1) $+ \lim_{M\to 1^+} \frac{3}{2}(x-1)^{\frac{2}{3}}\Big|_M^9$

$= \lim_{N\to 1^-} \frac{3}{2}\left((N-1)^{\frac{2}{3}} - 1\right) + \lim_{M\to 1^+} \frac{3}{2}\left(4 - (N-1)^{\frac{2}{3}}\right)$

$= \frac{3}{2}(0-1) + \frac{3}{2}(4-0)$

$= \frac{9}{2}$ (1)

ALL OTHER PARTS WORTH (1/2) EACH, EXCEPT THOSE MARKED (1)

Evaluate $\int (2+\sqrt{x})^8\,dx$.

SCORE: ___ / 5 PTS

$u = 2+\sqrt{x}$] EITHER ONE (1)
$x = (u-2)^2$

$dx = 2(u-2)\,du$

$\int u^8 \cdot 2(u-2)\,du = 2\int (u^9 - 2u^8)\,du$ ($\frac{1}{2}$, 1)

$= 2\left(\frac{1}{10}u^{10} - \frac{2}{9}u^9\right) + C$ (1)

$= \frac{1}{5}(2+\sqrt{x})^{10} - \frac{4}{9}(2+\sqrt{x})^9 + C$ (1, $\frac{1}{2}$)

Evaluate $\int_0^1 x^5 e^{-x^3}\,dx$.

SCORE: ___ / 5 PTS

$u = -x^3$ ⟷ $x=1 \to u=-1$, $x=0 \to u=0$

$du = -3x^2\,dx$

$dx = -\frac{1}{3x^2}\,du$

$x^5 e^{-x^3}\,dx = -\frac{1}{3}x^3 e^{-x^3}\,du$

$= -\frac{1}{3}ue^u\,du$

(1) $-\frac{1}{3}\int_0^{-1} ue^u\,du$ (1)

$= -\frac{1}{3}(ue^u - e^u)\Big|_0^{-1}$ (1)

$= -\frac{1}{3}(-e^{-1} - e^{-1} + 1)$

$= -\frac{1}{3}\left(1 - \frac{2}{e}\right)$ (1)

u	$+e^u$
1	$-e^u$
0	e^u

Evaluate $\int \frac{dx}{x+x\sqrt{x}}$.

SCORE: ___ / 5 PTS

(1) $x = u^2$

$dx = 2u\,du$

$\int \frac{2u\,du}{u^2+u^3}$

$= \int \frac{2}{u+u^2}\,du$ (1)

$= \int \left(\frac{2}{u} + \frac{-2}{1+u}\right)du$ (1)

$\frac{2}{u+u^2} = \frac{A}{u} + \frac{B}{1+u}$

$2 = A(1+u) + Bu$

$2 = A + (A+B)u$

$A = 2$

$A+B = 0 \to B = -2$

($\frac{1}{2}$) $2\ln|u| - 2\ln|1+u| + C$

(1) $2\ln\sqrt{x} - 2\ln(1+\sqrt{x}) + C$ ($\frac{1}{2}$) $= \ln x - 2\ln(1+\sqrt{x}) + C$